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Unitary IIB Matrix Model and the Dynamical Generation of the Space Time

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Abstract

We propose a unitary matrix model as a regularization of the IIB matrix model of Ishibashi-Kawai-Kitazawa-Tsuchiya (IKKT). The fermionic part is incorporated using the overlap formalism in order to avoid unwanted "doublers" while preserving the global gauge invariance. This regularization, unlike the one adopted by IKKT, has manifest $\mathrm{U}(1)^{10}$ symmetry, which corresponds to the ten-dimensional translational invariance of the space time. We calculate one-loop effective action around some typical BPS-saturated configurations in the weak coupling limit. We also discuss a possible scenario for the dynamical generation of the four-dimensional space time through spontaneous breakdown of the $\mathrm{U}(1)^{10}$ symmetry in the double scaling limit.

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1 Introduction

String theory has been studied as a natural candidate of the unified theory including quantum gravity. As the theory of everything, it should explain all the details of the standard model as low energy physics, such as the structure of the gauge group, the three generations of the matter, and even the space-time dimension. Perturbative study of string theory in the eighties revealed, however, that there are infinitely many perturbatively stable vacuua, and that we cannot make any physical prediction as to our present world unless we understand the nonperturbative effects. It is natural to expect that just as the confinement in QCD was understood only after lattice gauge theory appeared as a constructive definition of gauge theories [1], so must the true vacuum of superstring theory be understood once a constructive definition of string theories could be obtained. Recently, such a constructive definition of superstring theory using large N matrix model [2, 3] has been proposed. Solitonic objects known as D-branes [4, 5], which was focused in the context of string duality, plays an important role here. The basic idea is to quantize the lowest dimensional D-brane nonperturbatively, instead of string itself. It has been shown [6] that $1/N$ expansion of the model proposed by Ishibashi-Kawai-Kitazawa-Tsuchiya (IKKT) [3] gives the perturbation theory of type IIB superstring by reproducing the light-cone string field theory through Schwinger-Dyson equation. Above all, the way in which one should take the double scaling limit has been explicitly identified at least for sufficiently small string coupling constant. We may say that we are now at the stage to extract nonperturbative physics of superstring through this model.

As one of the most fundamental issues, let us consider how we can get the space-time dimension. Since the eigenvalues of the bosonic hermitian matrices of the IKKT model are interpreted as space-time coordinates of the D-objects, one possible scenario for the dynamical generation of the space time should be that in an appropriate double scaling limit the distribution of the eigenvalues degenerates to a four-dimensional hypersurface. Note here that the model before regularization possesses a symmetry under a transformation which shifts all the hermitian matrices by the identity matrix times a constant, which corresponds to the 10D translational invariance of the eigenvalue space. The degeneracy of the eigenvalue distribution, therefore, implies the spontaneous breakdown of the translational invariance. The translational invariance is also essential in reproducing the string perturbation theory [6]. A regularization, however, adopted by IKKT [3] in order to make the integration over the

bosonic hermitian matrices well-defined by requiring that the magnitude of the eigenvalues should be less than π/a, clearly violates the 10D translational invariance at the boundary of the eigenvalue space. A hope [6] might be that this does not cause any serious problem, since we take the cutoff a to zero in the end anyway, but it is certainly a flaw of this model.

We therefore consider in this paper, a regularization which preserves manifest 10D translational invariance. A natural candidate is to replace the bosonic Hermitian matrices by unitary matrices [1]. It is now the phases of the eigenvalues that are interpreted as the space-time coordinates. Thus the space time is naturally compactified to a ten-dimensional torus. The unitary matrix model can be considered as being reduced from a 10D lattice gauge theory. An obvious drawback of this kind of model, therefore, is that the continuous rotational invariance is broken down to the discrete one. Moreover a problem related to lattice fermion arises here. Namely when we consider the fermionic part naively, $2^{10} = 1024$ doublers will come out. Since the fermionic matrices are Majorana-Weyl spinor as a representation of the Lorentz group, we cannot easily decouple the unwanted doublers as is the case with ordinary lattice chiral gauge theories.

The problem of regularizing chiral gauge theories on the lattice has a long history and there are a lot of proposals made so far. The most promising one at present is the overlap formalism [8], which passed a number of tests [9]. The idea to apply this formalism to ten-dimensional $\mathcal{N} = 1$ (anomalous and non-renormalizable) super Yang-Mills theory, as a regularization of four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory via dimensional reduction, is mentioned in Ref. [10]. What we should consider here is essentially the large N reduced version of it. We note that although the overlap formalism as a regularization of ordinary lattice chiral gauge theories has a subtle problem with the local gauge invariance not being preserved on the lattice, its application to the present case is completely safe regarding this, since we do not have the local gauge invariance to take care of. The global gauge invariance, on the other hand, which is indeed one of the important symmetry of the model, is manifestly preserved within the formalism. The unitary matrix model thus defined has the $\mathrm{U}(1)^{10}$ symmetry, which corresponds to the 10D translational invariacne of the space time.

Another important symmetry of the IKKT model is the $\mathcal{N}$=2 supersymmetry regarding the eigenvalues of the bosonic Hermitian matrices as the space-time coordinates [3]. Note

[1]A unitary matrix model for M(atrix) Theory has been considered in Ref. [7] from a completely different motivation.

that the 10D translational invariance mentioned above gives a subgroup of the supersymmetry. This $\mathcal{N}$=2 supersymmetry comes from (1) the supersymmetry of 10D super Yang-Mills theory, combined with (2) the symmetry under constant shifts of the fermionic matrices. When we consider the unitary matrix model with the overlap formalism, we have (2) but not (1) unfortunately, and therefore, the $\mathcal{N}$=2 supersymmetry is not manifest. In the case of ordinary super Yang-Mills theories, supersymmetry is broken when we put the theory on the lattice, but is expected to be restored in the continuum limit with appropriate fine-tuning, as has been advocated in Ref. [11]. Fine-tuning can be avoided by the use of the overlap formalism due to the exact chiral symmetry for 4D $\mathcal{N} = 1$ super Yang-Mills theory [8] and due to the exact parity invariance for 3D $\mathcal{N} = 1$ super Yang-Mills theory [12]. This is based on the universality argument. Although the validity of the universality argument in matrix models is not clear at present, we naively expect the $\mathcal{N}$=2 supersymmetry to be restored in the double scaling limit without particular fine-tuning.

To summarize our strategy, we respect the $\mathrm{U}(1)^{10}$ symmetry or the 10D translational invariance as the most important symmetry of the model, while we give up the continuous 10D Lorentz invariance and the $\mathcal{N}$=2 supersymmetry as manifest symmetries and naively expect them to be restored in the double scaling limit. This we consider to be analogous to the case with lattice gauge theory [1], where gauge symmetry is respected and the continuous Poincare invariance is broken by the lattice regularization and expected to be restored only in the continuum limit.

We study our model through one-loop perturbative expansion in the weak coupling limit. We calculate one-loop effective action around classical vacua. In this case, the integration over the bosonic degrees of freedom is already done in Ref. [13] in the context of Eguchi-Kawai model [14], and it is known to give rise to the logarithmic attractive potential between two eigenvalue points [13]. This attractive potential is exactly cancelled by the contribution from the fermionic degrees of freedom when the eigenvalues are sufficiently close to each other in accordance with the IKKT model [3]. On the other hand, when the eigenvalues are farther apart, a weak attractive potential arises, unlike the case with IKKT model, where the potential is completely flat. Thus the $\mathrm{U}(1)^{10}$ symmetry of our model is spontaneously broken in the weak coupling limit, though much more mildly than in purely bosonic case, where the logarithmic attractive potential exists [13]. We also calculate the effective action around other BPS-saturated configurations which represent one D-string and two parallel D-strings.

Since the $\mathrm{U}(1)^{10}$ symmetry is expected to be restored in the strong coupling phase, there must be a phase transition. We consider that this phase transition provides a natural place to take the double scaling limit. Since our model preserves manifest $\mathrm{U}(1)^{10}$ symmetry, we can discuss the dynamical generation of the space time as the spontaneous breakdown of the $\mathrm{U}(1)^{10}$ symmetry. Roughly speaking, if the $\mathrm{U}(1)^{10}$ symmetry is broken down to $\mathrm{U}(1)^{D}$ for sufficiently small string coupling constant, our model is equivalent to a D-dimensional gauge theory due to the argument of Eguchi-Kawai [14]. However, according to Ref. [6], we have to take the double scaling limit at the weak coupling region. Since gauge theories in more than four dimensions should have an ultraviolet fixed point at the strong coupling regime, if any, the $\mathrm{U}(1)^{10}$ symmetry of our model is expected to be broken down to $\mathrm{U}(1)^{4}$ at least for sufficiently small string coupling constant. This gives a qualitative understanding of the dynamical origin of the space-time dimension 4.

This paper is organized as follows. In Section 2, we define our model and discuss the symmetries it possesses. In Section 3, we calculate the one-loop effective action around some typical BPS-saturated configurations including the one corresponding to the classical vacua, which shows that the $\mathrm{U}(1)^{10}$ symmetry is spontaneously broken in the weak coupling limit. In Section 4, we consider the double scaling limit of the present model. We argue that the phase transition accompanied with the spontaneous breakdown of the $\mathrm{U}(1)^{10}$ symmetry provides a natural place to take the double scaling limit and that the dynamical generation of the four-dimensional space time could be naturally expected to occur at the critical region. Section 5 is devoted to summary and future prospects.

2 The Unitary IIB Matrix Model and Its Symmetries

2.1 Review of the IKKT model

In this paper, we work in Euclidean space time. The IKKT model [3] is defined through the action

$$S = -\frac{1}{g^2}\left(\sum_{\mu\nu}\frac{1}{4}\mathrm{tr}[A_\mu, A_\nu]^2 + \frac{1}{2}\sum_{\mu}\mathrm{tr}\overline{\psi}\Gamma_\mu[A_\mu,\psi]\right), \tag{2.1}$$

which can be formally obtained by the zero-volume limit of 10D supersymmetric $\mathrm{U}(N)$ Yang-Mills theory. A_μ and ψ are $N\times N$ hermitian matrices, which transform as a vector and a Majorana-Weyl spinor respectively under 10D Lorentz group. Γ_μ are 10D gamma matrices

which satisfies

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu}. \tag{2.2}$$

Let us briefly review the symmetries of this model. The following three come from the 10D Lorentz invariance, $\mathcal{N}$=1 supersymmetry, and the global gauge invariance of the 10D supersymmetric Yang-Mills theory, respectively.

(i) 10D rotational invariance

(ii) zero-volume version of the supersymmetry

$$\begin{aligned}\delta\psi &= \frac{i}{2}\sum_{\mu\nu}[A_\mu, A_\nu]\Gamma_{\mu\nu}\epsilon, \\ \delta A_\mu &= i\bar{\epsilon}\Gamma_\mu\psi,\end{aligned} \tag{2.3}$$

where $\Gamma_{\mu\nu} = \frac{1}{2}(\Gamma_\mu\Gamma_\nu - \Gamma_\nu\Gamma_\mu)$.

(iii) global gauge invariance

$$\begin{aligned}\delta\psi &= i[\psi, \alpha], \\ \delta A_\mu &= i[A_\mu, \alpha],\end{aligned} \tag{2.4}$$

Note that although 10D supersymmetric Yang-Mills theory has gauge anomaly, this is not of much concern to us since the model after the zero-volume limit do not have *local* gauge invariance anyway. Note also that the IKKT model can actually be defined without any reference to 10D supersymmetric Yang-Mills theory, which cannot be considered as a consistent quantum field theory due to the gauge anomaly.

In addition to these symmetries, the following symmetries arise due to the zero-volume limit.

(iv) constant shift of the bosonic hermitian matrices

$$\begin{aligned}\delta\psi &= 0 \\ \delta A_\mu &= \alpha_\mu\end{aligned} \tag{2.5}$$

(v) constant shift of the fermionic hermitian matrices

$$\begin{aligned}\delta\psi = \xi \\ \delta A_\mu = 0\end{aligned} \tag{2.6}$$

As is shown in Ref. [3], the symmetries (ii) and (v), with the aid of the symmetry (iii) and the equations of motion give rise to the ten-dimensional $\mathcal{N} = 2$ supersymmetry, regarding

the eigenvalues of the bosonic hermitian matrices as the space-time coordinates. The symmetry (iv), which corresponds to the 10D translational invariance, gives a subgroup of this supersymmetry.

The integration over the bosonic hermitian matrices has to be regularized in order to be well-defined. In Ref. [3], the regularization was given by restricting the integration region to

$$|\text{eigenvalues of the } A_\mu| \leq \frac{\pi}{a}, \tag{2.7}$$

where a is the cutoff. Due to this regularization, the symmetries (i),(ii) and (iv) is violated at the boundary of the eigenvalue space. Therefore, the ten-dimensional $\mathcal{N}=2$ super Poincare invariance is not preserved in the strict sense. Although one might hope that this does not cause any serious problem since the cutoff is taken to zero with proper renormalization of the coupling constant in the end, it is certainly a flaw of this model.

2.2 Definition of the unitary IIB matrix model

We consider a regularization that has manifest symmetries which correspond to (iii), (iv), (v), and the discrete subgroup of the rotational invariance (i). Above all, as compared with the IKKT model, our model possesses manifest $\mathrm{U}(1)^{10}$ symmetry, which corresponds to the ten-dimensional translational invariance of the space time. We replace the bosonic hermitian matrices of the IKKT model by unitary matrices. The bosonic part of the action can be written as

$$S_b = -N\beta \sum_{\mu\nu} \mathrm{tr} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger. \tag{2.8}$$

The fermionic part could be naively written as

$$S_f = \sum_\mu \frac{1}{4} \left(\mathrm{tr}\overline{\psi}\Gamma_\mu U_\mu \psi U_\mu^\dagger - \mathrm{tr}\overline{\psi}\Gamma_\mu U_\mu^\dagger \psi U_\mu \right). \tag{2.9}$$

The total action $S_b + S_f$ can be formally obtained by reducing the 10D lattice gauge theory with naive Majorana-Weyl fermion to a unit cell. The naive fermion action in ordinary lattice gauge theories in D dimensions gives rise to 2^D doublers. The doublers can be seen as duplicated poles in the free fermion propagator in the momentum space. In the present model, we can see them by considering the perturbative expansion around the classical vacua

$$U_\mu = \mathrm{diag}(\mathrm{e}^{i\theta_{\mu 1}}, \mathrm{e}^{i\theta_{\mu 2}}, \cdots, \mathrm{e}^{i\theta_{\mu N}}). \tag{2.10}$$

The fermion propagator is given by

$$\langle \psi_{pq} \psi_{rs} \rangle = \left(\sum_{\mu} \Gamma_\mu \sin(\theta_{\mu p} - \theta_{\mu q}) \right)^{-1} \delta_{ps} \delta_{qr}. \tag{2.11}$$

The 2^{10} poles are given by $\theta_{\mu p} - \theta_{\mu q} = 0$ or π for each μ, while the only pole that appears in the IKKT model is given by $\theta_{\mu p} - \theta_{\mu q} = 0$. Thus, unless the $\mathrm{U}(1)^{10}$ is completely broken spontaneously and $\theta_{\mu p} - \theta_{\mu q} \approx 0$ dominates, the naive fermion action gives 1024 doublers, half of which being left handed and the rest being right handed. This is unacceptable since it violates the balance between the bosonic degrees of freedom and the fermionic degrees of freedom. Since the fermions are chiral, it is not easy to decouple the doublers, as is the case with ordinary lattice chiral gauge theories.

We overcome this problem in the present case, applying the overlap formalism, which has been developed to deal with lattice chiral gauge theories. We note that although the overlap formalism, as a regularization of lattice chiral gauge theory, has a subtle problem related to the local gauge invariance on the lattice, in the present application, we are completely free from this subtlety, since there is no local gauge invariance we have to respect. On the other hand, the overlap formalism preserves the 10D discrete rotational invariance and the global gauge invariance. Also we can identify a zero mode that corresponds to the fermion shift invariance.

We introduce the following notation for the adjoint representation of the $\mathrm{U}(N)$ group.

$$\begin{aligned} \psi_{pq} &= \sum_a (T^a)_{pq} \psi_a^{(\mathrm{A})} \\ \overline{\psi}_{pq} &= \sum_a (T^a)_{pq} \overline{\psi}_a^{(\mathrm{A})}, \end{aligned} \tag{2.12}$$

where T^a are the generators normalized as $\mathrm{tr}(T^a T^b) = \delta_{ab}$. Then the fermion action can be written as

$$\begin{aligned} S_f &= \sum_\mu \frac{1}{4} \left(\overline{\psi}^{(\mathrm{A})} \Gamma_\mu U_\mu^{(\mathrm{A})} \psi^{(\mathrm{A})} - \overline{\psi}^{(\mathrm{A})} \Gamma_\mu U_\mu^{(\mathrm{A})\dagger} \psi^{(\mathrm{A})} \right) \\ &= \sum_\mu \frac{1}{2} \overline{\psi}^{(\mathrm{A})} \Gamma_\mu D_\mu \psi^{(\mathrm{A})}, \end{aligned} \tag{2.13}$$

where we have introduced the adjoint link variable through

$$(U_\mu^{(\mathrm{A})})_{ab} = \mathrm{tr}(T^a U_\mu T^b U_\mu^\dagger) \tag{2.14}$$

and the reduced version of the covariant derivative through

$$D_\mu = \frac{1}{2} (U_\mu^{(\mathrm{A})} - U_\mu^{(\mathrm{A})\dagger}). \tag{2.15}$$

One can see that $U_\mu^{(\mathrm{A})}$ is real, *i.e.*, $(U_\mu^{(\mathrm{A})})^* = U_\mu^{(\mathrm{A})}$, which reflects the fact that the adjoint representation is a real representation.

We take a particular representation for the 10D gamma matrices in the following way. We first define 8D gamma matrices γ_i $(i = 1, 2, \cdots, 8)$ which satisfies $\{\gamma_i, \gamma_j\} = \delta_{ij}$. We take $\gamma_9 = \gamma_1\gamma_2\cdots\gamma_8$. Then the 10D gamma matrices is defined as

$$\begin{aligned} \Gamma_i &= \sigma_1 \otimes \gamma_i \qquad (i = 1, 2, \cdots, 9) \\ \Gamma_{10} &= \sigma_2 \otimes \mathbf{1}. \end{aligned} \tag{2.16}$$

Note that the chirality operator

$$\begin{aligned} \Gamma_{11} &= -i\Gamma_1\Gamma_2\cdots\Gamma_{10} \\ &= \sigma_3 \otimes \mathbf{1} \end{aligned} \tag{2.17}$$

is diagonal, which implies that the above construction gives a Weyl representation of the gamma matrices. Then the action for the fermionic matrices can be written as

$$\begin{aligned} S_f &= \sum_\mu \frac{1}{2}\overline{\psi}^{(\mathrm{A})}\Gamma_\mu D_\mu \psi^{(\mathrm{A})} \\ &= \sum_\mu \frac{1}{2}\psi^{(\mathrm{A})\dagger} i\Gamma_{10}\Gamma_\mu D_\mu \psi^{(\mathrm{A})} \\ &= \frac{1}{2}\psi^{(\mathrm{A})\dagger}(iD_{10} + \sigma_3 \otimes \sum_i \gamma_i D_i)\psi^{(\mathrm{A})} \\ &= \frac{1}{2}\begin{pmatrix} \psi_L^{(\mathrm{A})\dagger} & \psi_R^{(\mathrm{A})\dagger} \end{pmatrix} \begin{pmatrix} \sum_i \gamma_i D_i + iD_{10} & 0 \\ 0 & -\sum_i \gamma_i D_i + iD_{10} \end{pmatrix} \begin{pmatrix} \psi_L^{(\mathrm{A})} \\ \psi_R^{(\mathrm{A})} \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix} \psi_L^{(\mathrm{A})\dagger} & \psi_R^{(\mathrm{A})\dagger} \end{pmatrix} \begin{pmatrix} \mathbf{C} & 0 \\ 0 & \mathbf{C}^\dagger \end{pmatrix} \begin{pmatrix} \psi_L^{(\mathrm{A})} \\ \psi_R^{(\mathrm{A})} \end{pmatrix}, \end{aligned} \tag{2.18}$$

where we define the chiral operator as

$$\mathbf{C} = \sum_\mu \gamma_\mu D_\mu = \sum_\mu \gamma_\mu \frac{1}{2}(U_\mu^{(\mathrm{A})} - U_\mu^{(\mathrm{A})\dagger}), \tag{2.19}$$

and define γ_{10} as

$$\gamma_{10} = i\mathbf{1}. \tag{2.20}$$

Note that γ_i $(i = 1, 2, \cdots 9)$ are hermitian, while γ_{10} is anti-hermitian.

Let us first consider the fermion determinant for a 10D Weyl fermion in the present case through the overlap formalism [8]. We consider the many-body Hamiltonians defined as

$$\mathcal{H}_\pm = \begin{pmatrix} \alpha^\dagger & \beta^\dagger \end{pmatrix} \begin{pmatrix} \mathbf{B} \pm m & \mathbf{C} \\ \mathbf{C}^\dagger & -(\mathbf{B} \pm m) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}. \tag{2.21}$$

where $\{\alpha^\dagger, \alpha\}$ and $\{\beta^\dagger, \beta\}$ are sets of creation and annihilation operators which obey the canonical anti-commutation relations. The fermionic operators carry spinor indices as well as those for the adjoint representation of the U(N) group, which we have suppressed. $\mathbf{B}$ is defined through

$$\mathbf{B} = \sum_\mu \left\{ 1 - \frac{1}{2}(U_\mu^{(\mathrm{A})} + U_\mu^{(\mathrm{A})\dagger}) \right\}, \tag{2.22}$$

which plays the role of the Wilson term and eliminates the doublers [8]. We will see this explicitly in the next section. m is a constant which should be kept fixed within $0 < m < 1$ when one takes the double scaling limit.

We denote the ground states of the many-body Hamiltonians $\mathcal{H}_\pm$ as $|\pm\rangle_U$. We fix the U dependence of the phase of the states by imposing that ${}_1\langle\pm|\pm\rangle_U$ should be real positive. This is called the Wigner-Brillouin phase choice in Ref. [8]. We denote the ground states thus defined as $|\pm\rangle_U^{\mathrm{WB}}$. Then the fermion determinant for a single left-handed Weyl fermion is defined as ${}^{\mathrm{WB}}_{U}\langle -|+\rangle_U^{\mathrm{WB}}$.

We can see that the above formula can be decomposed into two Majorana-Weyl fermions following the steps taken in Ref. [10]. Note first that there exists a unitary matrix Ω which has the following property.

$$\Omega^\dagger \gamma_i \Omega = (\gamma_i)^t \qquad (i = 1, 2, \cdots, 10) \tag{2.23}$$

$$\Omega^t = \Omega. \tag{2.24}$$

For example, one can take the unitary matrix Ω to be identity, by choosing all the γ_i $(i = 1, 2, \cdots, 9)$ to be real, which corresponds to the case in which Γ_{10} and $i\Gamma_i$ $(i = 1, 2, \cdots, 9)$ defined through eq.(2.16) give a Majorana-Weyl representation of the 10D gamma matrices in Minkowski space. Using eqs. (2.23) and (2.24), one can show that

$$(\mathbf{C}\Omega)^t = -\mathbf{C}\Omega. \tag{2.25}$$

We make the following Bogoliubov transformation.

$$\begin{aligned} \alpha &= \frac{\xi + i\eta}{\sqrt{2}} \\ \beta &= \Omega \frac{\xi^{\dagger t} + i\eta^{\dagger t}}{\sqrt{2}}. \end{aligned} \tag{2.26}$$

Plugging this into eq.(2.21), we obtain

$$\begin{aligned} \mathcal{H}_\pm &= \frac{1}{2} \begin{pmatrix} \xi^\dagger & \xi^t \end{pmatrix} \begin{pmatrix} \mathbf{B} \pm m & \mathbf{C}\Omega \\ (\mathbf{C}\Omega)^\dagger & -(\mathbf{B} \pm m) \end{pmatrix} \begin{pmatrix} \xi \\ \xi^{\dagger t} \end{pmatrix} \\ &+ \frac{1}{2} \begin{pmatrix} \eta^\dagger & \eta^t \end{pmatrix} \begin{pmatrix} \mathbf{B} \pm m & \mathbf{C}\Omega \\ (\mathbf{C}\Omega)^\dagger & -(\mathbf{B} \pm m) \end{pmatrix} \begin{pmatrix} \eta \\ \eta^{\dagger t} \end{pmatrix}. \end{aligned} \tag{2.27}$$

Thus, ξ and η decouple and each term of the Hamiltonians corresponds to a Majorana-Weyl fermion.

The fermion determinant for a Majorana-Weyl fermion can, therefore, be defined as the overlap of the ground states of the two many-body Hamiltonians:

$$\mathcal{H}_{\pm}^{(\mathrm{Maj})} = \frac{1}{2} \begin{pmatrix} \xi^{\dagger} & \xi^{t} \end{pmatrix} \begin{pmatrix} \mathbf{B} \pm m & \mathbf{C}\Omega \\ (\mathbf{C}\Omega)^{\dagger} & -(\mathbf{B} \pm m) \end{pmatrix} \begin{pmatrix} \xi \\ \xi^{\dagger t} \end{pmatrix}. \tag{2.28}$$

Let us denote the ground states with the Wigner-Brillouin phase choice as $|\mathrm{M}\pm\rangle_U^{\mathrm{WB}}$. Since

$$|\pm\rangle_U^{\mathrm{WB}} = |\mathrm{M}\pm\rangle_U^{\mathrm{WB}} \otimes |\mathrm{M}\pm\rangle_U^{\mathrm{WB}}, \tag{2.29}$$

we have

$${}_U^{\mathrm{WB}}\langle -|+\rangle_U^{\mathrm{WB}} = \left({}_U^{\mathrm{WB}}\langle \mathrm{M}- \,|\mathrm{M}+\rangle_U^{\mathrm{WB}} \right)^2. \tag{2.30}$$

Thus the overlap formalism ensures that the fermion determinant of a Weyl fermion is the square of that of a Majorana-Weyl fermion, as it should be.

Our model can be defined through the following partition function.

$$Z = \int \prod_{\mu} [\mathrm{d}U_{\mu}] \mathrm{e}^{-S_b[U]} \; {}_U^{\mathrm{WB}}\langle \mathrm{M}- \,|\mathrm{M}+\rangle_U^{\mathrm{WB}}, \tag{2.31}$$

where $[\mathrm{d}U_{\mu}]$ denotes the Haar measure of the group integration over $\mathrm{U}(N)$. This gives a complete regularization of the IKKT model.

2.3 Symmetries of the unitary IIB matrix model

Let us turn to the symmetries of this model. As is proved in Ref. [8], the overlap has several properties which are expected as a sensible lattice regularization of a chiral determinant. In the present case, we have the invariance under 10D discrete rotational transformation and the global gauge transformation, among other things. The invariance of the magnitude of the overlap is essentially because the many-body Hamiltonians (2.28) can be formally derived from (10+1)D Majorana fermion with masses of opposite signs. The invariance of the phase comes from the fact that the gauge configuration $U_{\mu} = 1$, which is refered to in the Wigner-Brilloiun phase choice, is invariant under these transformations.

Furthermore, we have the $\mathrm{U}(1)^{10}$ symmetry:

$$U_{\mu} \to \mathrm{e}^{i\alpha_{\mu}} U_{\mu}. \tag{2.32}$$

This is because the boson action $S_b[U]$ and the Haar measure $[\mathrm{d}U_\mu]$ is invariant, and so is the fermion determinant given by the overlap, since it depends only on $U_\mu^{(\mathrm{A})}$, which is invariant under the above transformation.

The fermion shift symmetry has its counterpart in the naive fermion action (2.13), in which the fermion component $\{\psi_0^{(\mathrm{A})}, \bar{\psi}_0^{(\mathrm{A})}\}$ that corresponds to the generator $T^0 = \mathbf{1}$ does not appear, since

$$(U_\mu^{(\mathrm{A})})_{00} = 1; \qquad (U_\mu^{(\mathrm{A})})_{0a} = (U_\mu^{(\mathrm{A})})_{a0} = 0 \qquad \text{for } a \neq 0. \tag{2.33}$$

$\psi_0^{(\mathrm{A})}$, which is the trace part of the fermionic matrix, thus gives a zero mode.

We adopt this point of view in the overlap formalism, since here the action is not defined, but only the determinant after integrating over the fermionic matrices is given. Due to eq.(2.33), the component $\{\xi_0, \xi_0^\dagger\}$ that corresonds to the generator $T^0 = \mathbf{1}$ decouples from the others in the many-body Hamiltonian as

$$\begin{aligned} \mathcal{H}_\pm^{(\mathrm{Maj})} &= \frac{1}{2} \begin{pmatrix} \xi_0^\dagger & \xi_0^t \end{pmatrix} \begin{pmatrix} \pm m & 0 \\ 0 & \mp m \end{pmatrix} \begin{pmatrix} \xi_0 \\ \xi_0^{\dagger t} \end{pmatrix} + \cdots \\ &= \pm m \prod_{s=1}^{8} \xi_{0s}^\dagger \xi_{0s} + \cdots , \end{aligned} \tag{2.34}$$

where s represents the spinor index. Hence we can consider this part independently. In the subspace on which ξ_{0s} and $\xi_{0s}^\dagger$ act, we define the kinematical vacuum $|0\rangle$ by $\xi_{0s}|0\rangle = 0$. Then the ground states of the many-body Hamiltonians can be written as

$$\begin{aligned} |\mathrm{M}+\rangle_U^{\mathrm{WB}} &= |0\rangle \otimes \cdots \\ |\mathrm{M}-\rangle_U^{\mathrm{WB}} &= \prod_{s=1}^{8} \xi_{0s}^\dagger |0\rangle \otimes \cdots . \end{aligned} \tag{2.35}$$

We, therefore, have

$${}_U^{\mathrm{WB}}\langle \mathrm{M}- \, |\mathrm{M}+\rangle_U^{\mathrm{WB}} = 0. \tag{2.36}$$

Thus the trace part of the fermionic matrices gives a zero mode, which means that the overlap has a symmetry that corresponds to the fermion shift symmetry. In order to obtain a non-zero expectation value out of this model, we have to drop the zero-mode by hand, or equivalently, put them in the observables. In the rest of the paper, we assume that this is done implicitly, as has been done in the IKKT model [3].

To summarize, we have manifest symmetries corresponding to (iii),(iv) and (v). On the other hand, the 10D continuous rotational group (i) is broken down to a discrete one and

the zero-volume version of the supersymmetry (ii) is lost. Thus, the situation is exactly the same as with lattice formulation of supersymmetric Yang-Mills theories. As is advocated in Ref. [12], the overlap formalism can be used to obtain all the supersymmetric Yang-Mills theories in the continuum limit without fine-tuning. We expect here in the same spirit that 10D continuous rotational group (i) as well as the zero-volume version of the supersymmetry (ii) recovers in the double scaling limit without particular fine-tuning, resulting in a theory with 10D $\mathcal{N}=2$ super Poincare invariance.

2.4 The phase of the fermion determinant

The fermion determinant in the IKKT model is complex in general, and the phase depends on the bosonic matrices. As is mentioned in the Appendix of Ref. [3], however, it is real in the following two cases.

Case (1) : $P_\mu = 0$ at least for one direction.

Case (2) : $F_{\mu\nu} = [P_\mu, P_\nu] = 0$ for $1 \le \mu < \nu \le 10$.

Correspondingly in the present model, the fermion determinant is complex in general, and is real in the following two cases.

Case (1) : $U_\mu^{(\mathrm{A})} = 1$ at least for one direction.

Case (2) : $U_\mu^{(\mathrm{A})} U_\nu^{(\mathrm{A})} = U_\nu^{(\mathrm{A})} U_\mu^{(\mathrm{A})}$ for $1 \le \mu < \nu \le 10$.

We prove this in the following.

Let us first consider the Case (1). Without loss of generality, we can take $U_{10}^{(\mathrm{A})} = 1$. Then we have

$$\begin{aligned} \mathbf{C} &= \sum_{i=1}^{9} \gamma_i D_i = \sum_{i=1}^{9} \gamma_i \frac{1}{2}(U_i^{(\mathrm{A})} - U_i^{(\mathrm{A})\dagger}), \\ \mathbf{B} &= \sum_{i=1}^{9} \left\{ 1 - \frac{1}{2}(U_i^{(\mathrm{A})} + U_i^{(\mathrm{A})\dagger}) \right\}. \end{aligned} \tag{2.37}$$

The single-particle Hamiltonian

$$\mathrm{H}_\pm = \begin{pmatrix} \mathbf{B} \pm m & \mathbf{C}\Omega \\ (\mathbf{C}\Omega)^\dagger & -(\mathbf{B} \pm m) \end{pmatrix} \tag{2.38}$$

has the following symmetry.

$$\Sigma^\dagger \mathrm{H}_\pm \Sigma = \mathrm{H}_\pm^* \tag{2.39}$$

where

$$\Sigma = \begin{pmatrix} \Omega & 0 \\ 0 & \Omega^\dagger \end{pmatrix}. \tag{2.40}$$

Therefore, the many-body Hamiltonian (2.28) can be written as

$$\mathcal{H}_{\pm}^{(\mathrm{Maj})} = \frac{1}{2} \left(\begin{array}{cc} \xi^{\dagger} & \xi^{t} \end{array} \right) \mathrm{H}_{\pm} \left(\begin{array}{c} \xi \\ \xi^{\dagger t} \end{array} \right) = \frac{1}{2} \left(\begin{array}{cc} \eta^{\dagger} & \eta^{t} \end{array} \right) \mathrm{H}_{\pm}^{*} \left(\begin{array}{c} \eta \\ \eta^{\dagger t} \end{array} \right), \tag{2.41}$$

where $\eta = \Omega\xi$. The rest of the proof goes in exactly the same way as in Section III of Ref. [12], and we find that the fermion determinant defined through the overlap fromalism is real in this case. This result is naturally understood since the fermion determinant considered in this case can be regarded as that of a massless pseudo-Majorana fermion in nine dimensions [2], which is real. One can further show that if $U_{\mu}^{(\mathrm{A})} = 1$ at least for four directions, the overlap is not only real but also positive. This is because the fermion determinant can now be considered as that of massless Dirac fermion in six dimensions, which is real positive.

We next consider the Case (2). Here we show that the overlap for a Weyl fermion is real positive, which implies that the overlap for a Majorana-Weyl fermion is real due to the relation (2.30). Since $U_{\mu}^{(\mathrm{A})}$ commute with each other, they can be diagonalized simultaneously as follows.

$$U_{\mu}^{(\mathrm{A})} = V^{\dagger} \Lambda_{\mu} V, \tag{2.42}$$

where

$$(\Lambda_{\mu})_{ab} = \mathrm{e}^{i\theta_{\mu a}} \delta_{ab}. \tag{2.43}$$

Due to this, the many-body Hamiltonians (2.21) can be decomposed as

$$\begin{aligned} \mathcal{H}_{\pm} &= \sum_{a} \mathcal{H}_{\pm}^{a} \\ \mathcal{H}_{\pm}^{a} &= \left(\begin{array}{cc} \alpha_a'^{\dagger} & \beta_a'^{\dagger} \end{array} \right) \left(\begin{array}{cc} (b_a \pm m)\mathbf{1} & \mathbf{C}_a \\ \mathbf{C}_a^{\dagger} & -(b_a \pm m)\mathbf{1} \end{array} \right) \left(\begin{array}{c} \alpha_a' \\ \beta_a' \end{array} \right), \end{aligned} \tag{2.44}$$

where we redefine the fermionic operators as

$$\begin{aligned} \alpha_a' &= \sum_b V_{ab} \alpha_b \\ \beta_a' &= \sum_b V_{ab} \beta_b, \end{aligned} \tag{2.45}$$

and define

$$\mathbf{C}_a = i \sum_{\mu} \gamma_{\mu} \sin\theta_{\mu a} \tag{2.46}$$

$$b_a = \sum_{\mu} (1 - \cos\theta_{\mu a}) = 2 \sum_{\mu} \sin^2 \frac{\theta_{\mu a}}{2}. \tag{2.47}$$

[2] An application of the overlap formalism to gauge theories in odd dimensions has been studied in Ref. [15, 16], where the formalism can provide a parity invariant lattice regularization of massless Dirac fermion. An extension to Majorana fermion is given in Ref. [12]. A similar construction can be made for pseudo-Majorana fermion in 9D.

The Hamiltonians $\mathcal{H}_\pm^a$ can be further decomposed in the following way. Note first that $\sum_{i=1}^9 \gamma_i \sin\theta_{ia}$ is a hermitian matrix which has eigenvalues $\pm\sqrt{\sum_{i=1}^9 \sin^2\theta_{ia}}$, each of which has 8-fold degeneracy. Therefore, $\mathbf{C}_a$ can be diagonalized as

$$\mathbf{C}_a = W_a^\dagger \begin{pmatrix} z_a\mathbf{1} & 0 \\ 0 & z_a^*\mathbf{1} \end{pmatrix} W_a, \tag{2.48}$$

where

$$z_a = -\sin\theta_{0a} + i\sqrt{\sum_{i=1}^{9}\sin^2\theta_{ia}}. \tag{2.49}$$

Redefining the fermionic operators as

$$\begin{pmatrix} \phi_a \\ \chi_a \end{pmatrix} = W_a\alpha'_a \tag{2.50}$$

$$\begin{pmatrix} \psi_a \\ \omega_a \end{pmatrix} = W_a\beta'_a, \tag{2.51}$$

we obtain

$$\begin{aligned}\mathcal{H}_\pm^a &= \begin{pmatrix} \phi_a^\dagger & \psi_a^\dagger \end{pmatrix} \begin{pmatrix} (b_a\pm m)\mathbf{1} & z_a\mathbf{1} \\ z_a^*\mathbf{1} & -(b_a\pm m)\mathbf{1} \end{pmatrix} \begin{pmatrix} \phi_a \\ \psi_a \end{pmatrix} \\ &\quad + \begin{pmatrix} \chi_a^\dagger & \omega_a^\dagger \end{pmatrix} \begin{pmatrix} (b_a\pm m)\mathbf{1} & z_a^*\mathbf{1} \\ z_a\mathbf{1} & -(b_a\pm m)\mathbf{1} \end{pmatrix} \begin{pmatrix} \chi_a \\ \omega_a \end{pmatrix}. \end{aligned} \tag{2.52}$$

Thus the problem has reduced essentially to that of obtaining the overlap for the Hamiltonians

$$\widetilde{\mathcal{H}_\pm} = \begin{pmatrix} c^\dagger & d^\dagger \end{pmatrix} \begin{pmatrix} b\pm m & z \\ z^* & -(b\pm m) \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix}. \tag{2.53}$$

The ground states $\widetilde{|\pm\rangle}_U^{\rm WB}$ of $\widetilde{\mathcal{H}_\pm}$ with the Wigner-Brillouin phase choice can be obtained as

$$\widetilde{|+\rangle}_U^{\rm WB} = \frac{1}{\sqrt{2\epsilon^+(\epsilon^+ + \mu^+)}}\left(-zc^\dagger + (\epsilon^+ + \mu^+)d^\dagger\right)|0\rangle \tag{2.54}$$

$$\widetilde{|-\rangle}_U^{\rm WB} = \frac{1}{\sqrt{2\epsilon^-(\epsilon^- - \mu^-)}}\left((\epsilon^- - \mu^-)c^\dagger - z^*d^\dagger\right)|0\rangle, \tag{2.55}$$

where $\mu^\pm = b\pm m$ and $\epsilon^\pm = \sqrt{|z|^2 + (\mu^\pm)^2}$. $|0\rangle$ is the kinematical vacuum defined through $c|0\rangle = d|0\rangle = 0$. Note that $\widetilde{|+\rangle}_1^{\rm WB} = d^\dagger|0\rangle$ and $\widetilde{|-\rangle}_1^{\rm WB} = c^\dagger|0\rangle$, and ${}_1^{\rm WB}\widetilde{\langle\pm|\pm\rangle}_U^{\rm WB}$ is real positive, which ensures the Wigner-Brillouin phase choice. The overlap can be obtained as

$${}_U^{\rm WB}\widetilde{\langle-|+\rangle}_U^{\rm WB} = \frac{-z}{2\sqrt{\epsilon^+\epsilon^-}}\left(\sqrt{\frac{\epsilon^- - \mu^-}{\epsilon^+ + \mu^+}} + \sqrt{\frac{\epsilon^+ + \mu^+}{\epsilon^- - \mu^-}}\right). \tag{2.56}$$

Now returning to our problem, we find the overlap of the ground states of the many-body Hamiltonians (2.44) as

$$ {}^{\rm WB}_{U}\langle -|+\rangle^{\rm WB}_{U} = \prod_a \left[\frac{|z_a|^2}{4\epsilon_a^+\epsilon_a^-} \left(\sqrt{\frac{\epsilon_a^- - \mu_a^-}{\epsilon_a^+ + \mu_a^+}} + \sqrt{\frac{\epsilon_a^+ + \mu_a^+}{\epsilon_a^- - \mu_a^-}} \right)^2 \right]^8 , \tag{2.57} $$

where $\mu_a^\pm = b_a \pm m$ and $\epsilon_a^\pm = \sqrt{|z_a|^2 + (\mu_a^\pm)^2}$. Thus the fermion determinant for a Weyl fermion defined through the overlap formalism is real positive. This completes the proof that the overlap for a Majorana-Weyl fermion is real in the Case (2).

We can further show that the overlap for a Majorana-Weyl fermion is real positive in the Case (2) through the following argument. We first note that since $\mathcal{H}_\pm^{(\rm Maj)}$ depend on $U_\mu^{(\rm A)}$ continuously, we can choose the phases of the ground states $|{\rm M}\pm\rangle_U$ so that the states depend on $U_\mu^{(\rm A)}$ continuously including their phases. We define the ground states that obey the Wigner-Brillouin phase choice as

$$ |{\rm M}\pm\rangle^{\rm WB}_{U} = {\rm e}^{i\theta_\pm(U)} |{\rm M}\pm\rangle_U . \tag{2.58} $$

Since

$$ {}^{\rm WB}_{1}\langle {\rm M}\pm\, |{\rm M}\pm\rangle^{\rm WB}_{U} = {\rm e}^{i(\theta_\pm(U)-\theta_\pm(1))}\ \ {}_1\langle {\rm M}\pm\, |{\rm M}\pm\rangle_U , \tag{2.59} $$

the requirement that the left-hand side should be real positive determines ${\rm e}^{i\theta_\pm(U)}$ except when ${}_1\langle {\rm M}\pm\, |{\rm M}\pm\rangle_U = 0$. For configurations such that ${}_1\langle {\rm M}\pm\, |{\rm M}\pm\rangle_U \neq 0$, the phase factors ${\rm e}^{i\theta_\pm(U)}$ and therefore the overlap

$$ {}^{\rm WB}_{U}\langle {\rm M}-\, |{\rm M}+\rangle^{\rm WB}_{U} = {\rm e}^{i(\theta_+(U)-\theta_-(U))}\ \ {}_U\langle {\rm M}-\, |{\rm M}+\rangle_U \tag{2.60} $$

are continuous functions of $U_\mu^{(\rm A)}$.

The phase of ${}^{\rm WB}_{U}\langle {\rm M}-\, |{\rm M}+\rangle^{\rm WB}_{U}$ is therefore a continuous function of $U_\mu^{(\rm A)}$ except when

$$ \begin{aligned} {}^{\rm WB}_{1}\langle {\rm M}\pm\, |{\rm M}\pm\rangle^{\rm WB}_{U} &= 0, \\ {}^{\rm WB}_{U}\langle {\rm M}-\, |{\rm M}+\rangle^{\rm WB}_{U} &= 0. \end{aligned} \tag{2.61} $$

Let us refer to such $U_\mu^{(\rm A)}$ as *singular* configurations. In the present case, $U_\mu^{(\rm A)}$ is labeled by $\{\theta_{\mu a}\}$ and the singular configurations correspond to the $\{\theta_{\mu a}\}$ for which there exists an a such that $\theta_{\mu a} = 0$ or π for all μ. Note that any two configurations $\{\theta_{\mu a}\}$ and $\{\theta'_{\mu a}\}$ which are not themselves singular can be connected continuously without passing through singular configurations. This means that the overlap can be chosen to be real positive for all $\{\theta_{\mu a}\}$ [3].

[3] Correspondingly in the IKKT model, a similar argument using continuity can be used to show that the fermion determinant is real positive in the Case (2).

Therefore we can write the overlap for a Majorana-Weyl fermion as

$$ {}^{\mathrm{WB}}_{U}\langle \mathrm{M}- | \mathrm{M}+\rangle^{\mathrm{WB}}_{U} = \prod_a \left[\frac{|z_a|^2}{4\epsilon_a^+\epsilon_a^-} \left(\sqrt{\frac{\epsilon_a^- - \mu_a^-}{\epsilon_a^+ + \mu_a^+}} + \sqrt{\frac{\epsilon_a^+ + \mu_a^+}{\epsilon_a^- - \mu_a^-}} \right)^2 \right]^4 . \tag{2.62} $$

Note also that the result depends only on $b_a = 2\sum_\mu \sin^2 \frac{\theta_{\mu a}}{2}$ and $|z_a|^2 = \sum_\mu \sin^2 \theta_{\mu a}$, which ensures the 10D discrete rotational invariance. The explicit formula we have obtained here will be used in the next section to calculate the one-loop effective action around some typical BPS-saturated configurations.

3 One-loop effective action for BPS-saturated backgrounds

In this section we calculate one-loop effective action around some typical BPS-saturated backgrounds such as N D-instantons, one D-string and two parallel D-strings. The classical equation of motion for the bosonic part can be obtained by substituting $U_\mu = \bar{U}_\mu \mathrm{e}^{ia_\mu}$ in the action (2.8) and requiring that the first order terms in a_μ cancel. The result reads

$$ \sum_\nu (\bar{U}_\nu W_{\mu\nu} \bar{U}_\nu^\dagger - W_{\mu\nu}) - \mathrm{h.c.} = 0 \qquad \text{for all } \mu, \tag{3.1} $$

where we define

$$ W_{\mu\nu} = \bar{U}_\mu^\dagger \bar{U}_\nu^\dagger \bar{U}_\mu \bar{U}_\nu . \tag{3.2} $$

By writing $\bar{U}_\mu = \mathrm{e}^{iA_\mu}$ and expanding in the eq. (3.1) in terms of A_μ, we find to the leading order that

$$ \sum_\nu [A_\nu, [A_\nu, A_\mu]] = 0 \qquad \text{for all } \mu, $$

which agrees with the equation of motion of the IKKT model [3]. One-loop effective action is calculated using the background field method. The contribution from the fermionic part is given by the overlap for the background configuration $\bar{U}_\mu$.

We calculate the contribution from the bosonic part in the following. The calculation can be done by a slight generalization of the one given in Ref. [13]. We expand the bosonic unitary matrix around the background as

$$ U_\mu = \bar{U}_\mu \mathrm{e}^{ia_\mu}, $$

where a_μ is a hermitian matrix which represents the quantum fluctuation. Putting this into the bosonic part of the action (2.8) and expanding it up to the second order of a_μ, we obtain

$$\begin{aligned} S_b^{(2)} &= -N\beta\sum_{\mu\nu}\Big[\mathrm{tr}\left\{(\bar{U}_\nu^\dagger a_\mu \bar{U}_\nu - a_\mu)(\bar{U}_\mu^\dagger a_\nu \bar{U}_\mu - a_\nu)W_{\mu\nu}\right\} - \mathrm{tr}\left\{[a_\mu, a_\nu]W_{\mu\nu}\right\} \\ &\quad -\frac{1}{2}\mathrm{tr}\left\{(\bar{U}_\nu^\dagger a_\mu \bar{U}_\nu - a_\mu)(\bar{U}_\nu^\dagger a_\mu \bar{U}_\nu - a_\mu)(W_{\mu\nu}+W_{\nu\mu})\right\} \\ &\quad -\frac{1}{2}\mathrm{tr}\left\{a_\mu[\bar{U}_\nu^\dagger a_\mu \bar{U}_\nu, (W_{\mu\nu}-W_{\nu\mu})]\right\}\Big]. \end{aligned} \tag{3.3}$$

We use the following gauge fixing term

$$S_{g.f.} = N\beta\mathrm{tr}\left\{\sum_\mu[\bar{U}_\mu^\dagger, U_\mu](\sum_\nu[\bar{U}_\nu^\dagger, U_\nu])^\dagger\right\}. \tag{3.4}$$

Expanding this in term of a_μ up to the second order, we obtain

$$S_{g.f.}^{(2)} = N\beta\sum_{\mu\nu}\mathrm{tr}\left\{(\bar{U}_\mu^\dagger a_\mu \bar{U}_\mu - a_\mu)(\bar{U}_\nu^\dagger a_\nu \bar{U}_\nu - a_\nu)\right\}. \tag{3.5}$$

The corresponding Faddeev-Popov ghost term is given by

$$S_{F.P.} = N\beta\sum_\mu \mathrm{tr}\left\{[b, \bar{U}_\mu^\dagger][U_\mu, c]\right\}. \tag{3.6}$$

Up to the second order of the quantum fields, we have

$$S_{F.P.}^{(2)} = -N\beta\sum_\mu\left[\mathrm{tr}\left\{b(\bar{U}_\mu c\bar{U}_\mu^\dagger - c)\right\} + \mathrm{tr}\left\{b(\bar{U}_\mu^\dagger c\bar{U}_\mu - c)\right\}\right] \tag{3.7}$$

The background configurations $\bar{U}_\mu$ we consider in the following correspond to BPS-saturated configurations for which $W_{\mu\nu} = \mathrm{e}^{i\alpha_{\mu\nu}}$, where $\alpha_{\mu\nu}$ is either 0 or a c-number of order $\frac{1}{N}$. Therefore $S_b^{(2)}$ can be written as

$$\begin{aligned} S_b^{(2)} &= -N\beta\sum_{\mu\nu}\cos\alpha_{\mu\nu}\Big[\mathrm{tr}\left\{(\bar{U}_\mu^\dagger a_\mu \bar{U}_\mu - a_\mu)(\bar{U}_\nu^\dagger a_\nu \bar{U}_\nu - a_\nu)\right\} \\ &\quad - \mathrm{tr}\left\{(\bar{U}_\nu^\dagger a_\mu \bar{U}_\nu - a_\mu)(\bar{U}_\nu^\dagger a_\mu \bar{U}_\nu - a_\mu)\right\}\Big]. \end{aligned} \tag{3.8}$$

Neglecting $O(\frac{1}{N^2})$, we can drop the factor $\cos\alpha_{\mu\nu}$. Putting together, we obtain the total action for the bosonic part up to the second order of the quantum fields as

$$\begin{aligned} S^{(2)} &= S_b^{(2)} + S_{g.f.}^{(2)} + S_{F.P.}^{(2)} \\ &= N\beta\sum_{\mu\nu}\mathrm{tr}\left\{(\bar{U}_\nu^\dagger a_\mu \bar{U}_\nu - a_\mu)(\bar{U}_\nu^\dagger a_\mu \bar{U}_\nu - a_\mu)\right\} \\ &\quad -N\beta\sum_\mu\left[\mathrm{tr}\left\{b(\bar{U}_\mu c\bar{U}_\mu^\dagger - c)\right\} + \mathrm{tr}\left\{b(\bar{U}_\mu^\dagger c\bar{U}_\mu - c)\right\}\right]. \end{aligned} \tag{3.9}$$

Introducing the adjoint notation through

$$(a_\mu)_{pq} = \sum_a (T^a)_{pq} (a_\mu^{(\mathrm{A})})_a, \tag{3.10}$$

and similarly for the ghost fields, we rewrite the eq. (3.9) as

$$\begin{aligned} S^{(2)} &= N\beta \sum_\mu a_\mu^{(\mathrm{A})t} \sum_\rho (2 - \bar{U}_\rho^{(\mathrm{A})} - \bar{U}_\rho^{(\mathrm{A})\dagger}) a_\mu^{(\mathrm{A})} \\ &\quad + N\beta b^{(\mathrm{A})t} \sum_\rho (2 - \bar{U}_\rho^{(\mathrm{A})} - \bar{U}_\rho^{(\mathrm{A})\dagger}) c^{(\mathrm{A})}, \end{aligned} \tag{3.11}$$

where the adjoint link variable $\bar{U}_\mu^{(\mathrm{A})}$ is defined by $(\bar{U}_\mu^{(\mathrm{A})})_{ab} = \mathrm{tr}(T^a \bar{U}_\mu T^b \bar{U}_\mu^\dagger)$.

The effective action can be obtained as

$$\begin{aligned} W_b &= -\log \int \mathrm{d}a_\mu \mathrm{d}b\ \mathrm{d}c\ \mathrm{e}^{-S^{(2)}} \\ &= 4 \log \det (\sum_\mu (2 - \bar{U}_\mu^{(\mathrm{A})} - \bar{U}_\mu^{(\mathrm{A})\dagger})). \end{aligned} \tag{3.12}$$

Since the $\bar{U}_\mu^{(\mathrm{A})}$ commute with one another in the present case, they can be diagonalized simultaneously, and we denote the diagonal elements by $\lambda_{\mu a} = \mathrm{e}^{i\theta_{\mu a}}$. Then we can write our result for the bosonic part as

$$W_b = 4 \sum_a \log(\sum_\mu 2 \sin^2 \frac{\theta_{\mu a}}{2}). \tag{3.13}$$

The fermionic part $W_f = -\log\ {}^{\mathrm{WB}}_{\bar{U}}\langle \mathrm{M}- | \mathrm{M}+ \rangle^{\mathrm{WB}}_{\bar{U}}$ is given by the formula obtained in the previous section, since the present case corresponds to the Case (2). Adding the bosonic part and the fermionic part, we finally obtain the total one-loop effective action as

$$\begin{aligned} W_{tot} &= W_b + W_f \\ &= 4 \sum_a \left[\log b_a - \log \left\{ \frac{\kappa_a}{\epsilon_a^+ \epsilon_a^-} \left(\sqrt{\frac{\epsilon_a^- - \mu_a^-}{\epsilon_a^+ + \mu_a^+}} + \sqrt{\frac{\epsilon_a^+ + \mu_a^+}{\epsilon_a^- - \mu_a^-}} \right)^2 \right\} \right], \end{aligned} \tag{3.14}$$

where $\mu_a^\pm = b_a \pm m$, $\epsilon_a^\pm = \sqrt{\kappa_a + (\mu_a^\pm)^2}$, $b_a = 2 \sum_\mu \sin^2 \frac{\theta_{\mu a}}{2}$ and $\kappa = \sum_\mu \sin^2 \theta_{\mu a}$. For each a such that $\theta_{\mu a} = 0$ for all μ, a zero mode appears both in the bosonic part and in the fermionic part. We should drop them by hand when we define the effective action. In the summation over a in eq. (3.14), therefore, all such a's should be excluded. As can be seen from the above derivation, ten times the number of zero modes is equal to the number of flat directions of the bosonic action up to gauge transformation. Note also that the a that corresponds to $T^0 = \mathbf{1}$ always gives a zero mode as is explained in section 2.3. The corresponding flat directions are $\bar{U}_\mu \to \bar{U}_\mu \mathrm{e}^{i\alpha_\mu}$.

In the following subsections, we consider specific BPS-saturated backgrounds that corresponds to N D-instantons, one D-string, and two parallel D-strings. Since we have already obtained the general formula, all we have to do is to diagonalize $\bar{U}_\mu^{(\mathrm{A})}$ explicitly for each given background configuration $\bar{U}_\mu$.

3.1 Effective action for classcal vacua (N D-instantons)

In this subsection we consider classical vacua as the background configuration. The classical vacua, or the global minima of the bosonic action (2.8) is given by configurations $\bar{U}_\mu$ which satisfy $\bar{U}_\mu \bar{U}_\nu = \bar{U}_\nu \bar{U}_\mu$. Up to gauge transformation, we can take the following particular configurations.

$$\bar{U}_\mu = \mathrm{diag}(\mathrm{e}^{i\theta_{\mu 1}}, \mathrm{e}^{i\theta_{\mu 2}}, \cdots, \mathrm{e}^{i\theta_{\mu N}}), \tag{3.15}$$

where $\theta_{\mu I}$ $(I = 1, 2, \cdots, N)$ are arbitrary parameters. These configurations represent N D-instantons, and the $\theta_{\mu I}$ are interpreted as 10D coordinates of the I-th D-instanton.

In order to calculate $(\bar{U}_\mu^{(\mathrm{A})})_{ab}$, we introduce the following explicit form for the generators T^a.

$$\begin{aligned}
(X^{(I,J),1})_{pq} &= \frac{1}{\sqrt{2}}(\delta_{pI}\delta_{qJ} + \delta_{pJ}\delta_{qI}) &(3.16)\\
(X^{(I,J),2})_{pq} &= \frac{1}{\sqrt{2}}(-i\delta_{pI}\delta_{qJ} + i\delta_{pJ}\delta_{qI}) &(3.17)\\
(Y^k)_{pq} &= \delta_{pk}\delta_{qk} &(3.18)
\end{aligned}$$

where the indices I, J and k run over $1 \le I < J \le N$ and $1 \le k \le N$, respectively.

$\bar{U}_\mu^{(\mathrm{A})}$ is already block diagonal and has non-vanishing elements only for the following choice of a and b.

(i) $T^a = X^{(I,J),\alpha}$ and $T^b = X^{(I,J),\beta}$

$(\bar{U}_\mu^{(\mathrm{A})})_{ab} = (R_\mu^{(I,J)})_{\alpha\beta}$, where the matrix $R_\mu^{(I,J)}$ is given as

$$R_\mu^{(I,J)} = \begin{pmatrix} \cos(\theta_{\mu I} - \theta_{\mu J}) & -\sin(\theta_{\mu I} - \theta_{\mu J}) \\ \sin(\theta_{\mu I} - \theta_{\mu J}) & \cos(\theta_{\mu I} - \theta_{\mu J}) \end{pmatrix}. \tag{3.19}$$

Therefore, the $(\bar{U}_\mu^{(\mathrm{A})})_{ab}$ can be diagonalized simultaneously and the diagonal elements $\lambda_\mu^{(I,J),\alpha}$ can be given as

$$\lambda_\mu^{(I,J),1} = \mathrm{e}^{i(\theta_{\mu I} - \theta_{\mu J})}, \qquad \lambda_\mu^{(I,J),2} = \mathrm{e}^{-i(\theta_{\mu I} - \theta_{\mu J})}. \tag{3.20}$$

(ii) $T^a = Y^k$ and $T^b = Y^k$

$(\bar{U}_\mu)_{ab} = 1$. These give zero modes, which we subtract by hand. Note that there are N zero modes corresponding to $10N$ flat directions $\theta_{\mu I}$ for moving each of the N D-instantons in arbitrary direction.

Substituting the $\lambda_\mu^{(I,J),\alpha}$ into the formula (3.14), we obtain

$$
\begin{aligned}
W^{(D-inst)} &= \sum_{I<J} w_{tot}(I,J) \\
w_{tot}(I,J) &= w_b(I,J) + w_f(I,J) \\
w_b(I,J) &= 8 \log b_{IJ} \\
w_f(I,J) &= -8 \log \left\{ \frac{\kappa_{IJ}}{\epsilon_{IJ}^{+}\epsilon_{IJ}^{-}} \left(\sqrt{\frac{\epsilon_{IJ}^{-} - \mu_{IJ}^{-}}{\epsilon_{IJ}^{+} + \mu_{IJ}^{+}}} + \sqrt{\frac{\epsilon_{IJ}^{+} + \mu_{IJ}^{+}}{\epsilon_{IJ}^{-} - \mu_{IJ}^{-}}} \right)^2 \right\},
\end{aligned}
\tag{3.21}
$$

where we define $\mu_{IJ}^{\pm} = b_{IJ} \pm m$, $\epsilon_{IJ}^{\pm} = \sqrt{\kappa_{IJ} + (\mu_{IJ}^{\pm})^2}$, $b_{IJ} = 2\sum_\mu \sin^2\left(\frac{\theta_{\mu I} - \theta_{\mu J}}{2}\right)$ and $\kappa_{IJ} = \sum_\mu \sin^2(\theta_{\mu I} - \theta_{\mu J})$. The result for the bosonic part agrees with the one obtained in Ref. [13].

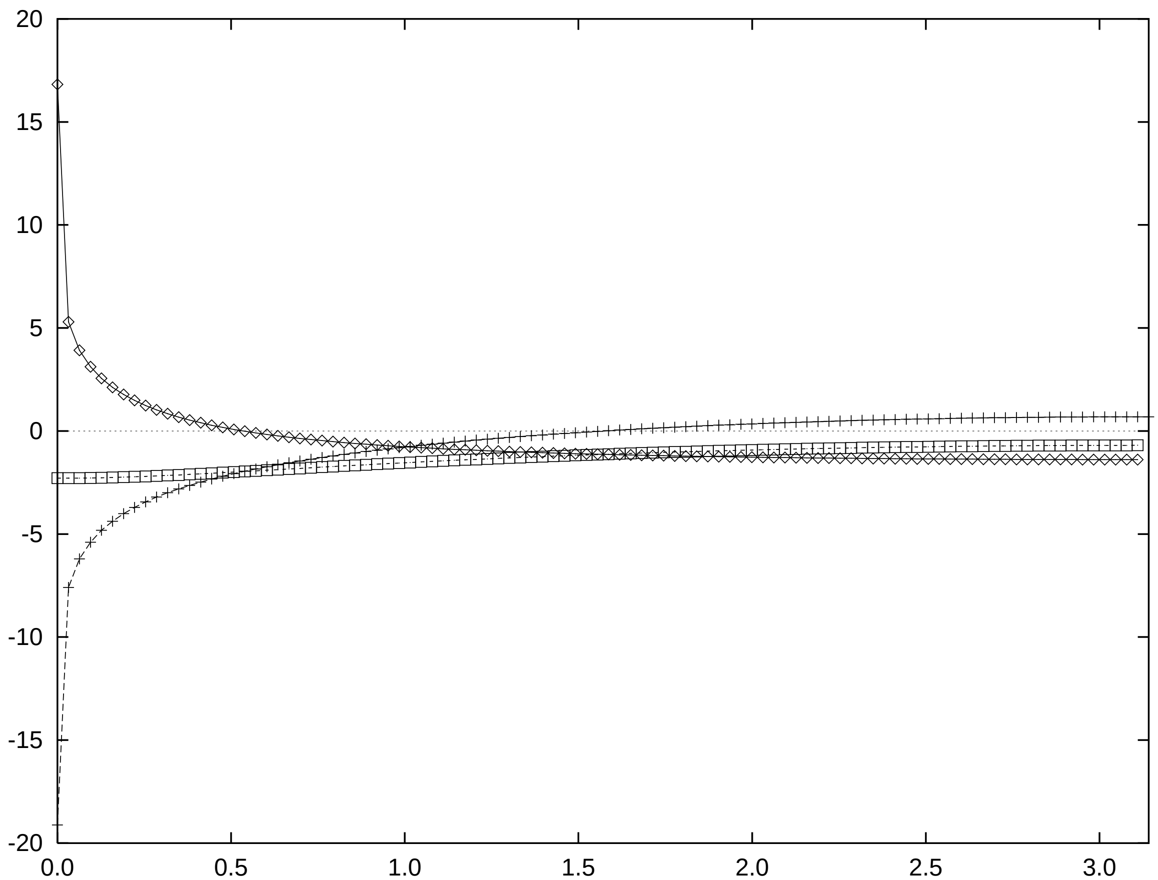

Figure : The two-body potential between two instantons is ploted against $|\theta_{1I} - \theta_{1J}|$. The parameter m is taken to be $m = 0.9$. Crosses, diamonds and squares represent w_b, w_f and w_{tot} respectively.

The effective action is written as a sum of the two-body potential $w_{tot}(I,J)$ between all the pairs of D-instantons. In the Figure, we plot the two-body potential $w_{tot}(I,J)$ for the

configuration with $\theta_{iI} - \theta_{iJ} = 0$ $(i = 2, 3, \cdots, 10)$ as a function of $|\theta_{1I} - \theta_{1J}|$. The parameter m is taken to be 0.9, but the result is qualitatively the same for any m within $0 < m < 1$. Note that the bosonic part gives an attractive potential, while the fermionic part gives a repulsive one, and the logarithmic singularity at $|\theta_{1I} - \theta_{1J}| \approx 0$ in $w_b(I,J)$ and $w_f(I,J)$ cancel each other. Note also that another logarithmic singularity that we would have in $w_f(I,J)$ at $|\theta_{1J} - \theta_{1J}| \approx \pi$ if the naive ferimon action (2.13) were used is absent thanks to the existence of b_{IJ} in $\mu^{\pm}_{IJ}$. This means that the doublers are successfully eliminated by the use of the overlap formalism. The total potential $w_{tot}(I,J)$ is monotonously increasing for $0 \leq |\theta_{1I} - \theta_{1J}| \leq \pi$ and the behavior at small dimstance is given by

$$w_{tot}(I,J) \sim \text{const.} + 8\left(\frac{1}{m^2} + \frac{1}{4}\right)|\theta_{1I} - \theta_{1J}|^2. \tag{3.22}$$

In the large N limit, the D-instantons are attracted to each other and the $\mathrm{U}(1)^{10}$ symmetry is spontaneously broken. Note, however, that the net attractive potential is much weaker than the one for the purely bosonic case, which has logarithmic singularity at the origin.

When one interprets the $\theta_{\mu I}$ as 10D coordinates of the I-th D-instanton, the physical coordinates should be given by

$$X_{\mu I} = \frac{\theta_{\mu I}}{a}, \tag{3.23}$$

where a is a cutoff parameter which should be taken to zero keeping Na^2 fixed when one takes the large N limit according to Ref. [6]. The two-body potential can be written in terms of the physical coordinates $X_{\mu I}$ as

$$w_{tot}(I,J) \sim \text{const.} + 8\left(\frac{1}{m^2} + \frac{1}{4}\right)a^2 \sum_\mu |X_{\mu I} - X_{\mu J}|^2 + O(a^4), \tag{3.24}$$

and one might be tempted to consider that the potential is independent of $\sum_\mu |X_{\mu I} - X_{\mu J}|^2$ in the $a \to 0$ limit. We should note, however, that the number of D-instantons N goes to infinity as a goes to zero. If we introduce the density function $\rho(X)$ as

$$\rho(X) = \frac{1}{N}\sum_{I=1}^{N}\prod_{\mu}^{10}\delta(X_\mu - X_{\mu I}), \tag{3.25}$$

the effective action can be written as

$$\begin{aligned} W^{(D-inst)} &= N^2 \int \mathrm{d}X_\mu\, \mathrm{d}Y_\mu\, \rho(X) 8\left(\frac{1}{m^2} + \frac{1}{4}\right)a^2 \sum_\mu |X_\mu - Y_\mu|^2 \rho(Y) \\ &= \text{const.}\ N \int \mathrm{d}X_\mu\, \mathrm{d}Y_\mu\, \rho(X) \sum_\mu |X_\mu - Y_\mu|^2 \rho(Y), \end{aligned} \tag{3.26}$$

where we have used the fact that we fix Na^2, when we take the double scaling limit [6]. Therefore, in the large N limit, the above attractive potential dominates and the density function $\rho(X)$ approaches $\prod_\mu \delta(X_\mu)$. This means that although the two-body force between two D-instantons placed at a finite physical distance becomes weaker and weaker in the large N limit, the increase in the number of D-instantons results in a collapse of the distribution function. In this sense, our model is not equivalent to the IKKT model at least in the weak coupling limit. We discuss further on this point in Section 4.

3.2 Effective action for a D-string

We next consider the background configuration which represents one D-string.

$$\bar{U}_1 = \Gamma_1; \qquad \bar{U}_2 = \Gamma_2; \qquad \bar{U}_i = \mathbf{1} \quad \text{for} \quad i = 3, 4, \cdots, 10, \tag{3.27}$$

where Γ_1 and Γ_2 are unitary matrices which satisfy the following algebra.

$$\Gamma_1 \Gamma_2 = \mathrm{e}^{\frac{2\pi i}{N}} \Gamma_2 \Gamma_1. \tag{3.28}$$

It is known that the solution to this algebra (3.28) is unique up to the gauge transformation $\Gamma_j' = g\Gamma_j g^\dagger$ and the U(1) transformation $\Gamma_j' = \mathrm{e}^{i\theta_j}\Gamma_j$, where $j = 1, 2$. Since our model is invariant under these transformations, we take the following specific matrices without loss of generality.

$$\begin{aligned} (\Gamma_1)_{pq} &= \mathrm{e}^{\frac{2\pi i}{N} p} \delta_{pq} \\ (\Gamma_2)_{pq} &= \delta_{\overline{p+1},q} \end{aligned} \tag{3.29}$$

We define the symbol $\overline{p}$ by

$$\overline{p} = \begin{cases} p & \text{for} \quad l \leq p \leq N \\ p - N & \text{for} \quad N+1 \leq p \leq 2N. \end{cases} \tag{3.30}$$

$\bar{U}^{(\mathrm{A})}_\mu$ can be given by

$$\begin{aligned} (\bar{U}^{(\mathrm{A})}_1)_{ab} &= (T^a)_{pq} (T^b)_{qp} \mathrm{e}^{\frac{2\pi i}{N}(q-p)} \\ (\bar{U}^{(\mathrm{A})}_2)_{ab} &= (T^a)_{pq} (T^b)_{\overline{q+1},\overline{p+1}} \\ (\bar{U}^{(\mathrm{A})}_i)_{ab} &= \delta_{ab} \qquad \text{for} \quad i = 3, 4, \cdots, 10. \end{aligned} \tag{3.31}$$

In view of this, we introduce the following notation for the X-type generators.

$$\begin{aligned} X^{Kn1} &= X^{(n,\overline{K+n}),1}, \quad i.e., \quad (X^{Kn1})_{pq} = \frac{1}{\sqrt{2}}(\delta_{pn}\delta_{q,\overline{K+n}} + \delta_{p,\overline{K+n}}\delta_{qn}) \\ X^{Kn2} &= X^{(n,\overline{K+n}),2}, \quad i.e., \quad (X^{Kn2})_{pq} = \frac{1}{\sqrt{2}}(-i\delta_{pn}\delta_{q,\overline{K+n}} + i\delta_{p,\overline{K+n}}\delta_{qn}) \end{aligned} \tag{3.32}$$

where $n = 1, 2, \cdots, N$ and $K = 1, 2, \cdots, \frac{N-1}{2}$. We assumed N to be an odd number for simplicity. One finds that $\bar{U}_\mu^{(\mathrm{A})}$ is already block-diagonal and has non-vanishing elements only for the following choice of a and b.

(i) $T^a = X^{Kn\alpha}$ and $T^b = X^{Km\beta}$

$$\begin{aligned} (\bar{U}_1^{(\mathrm{A})})_{ab} &= (R_K)_{\alpha\beta}\delta_{nm} \\ (\bar{U}_2^{(\mathrm{A})})_{ab} &= \delta_{\alpha\beta}\delta_{\overline{n+1},m} \\ (\bar{U}_i^{(\mathrm{A})})_{ab} &= \delta_{\alpha\beta}\delta_{nm} \quad \text{for} \quad i = 3, 4, \cdots, 10, \end{aligned} \tag{3.33}$$

where the matrix R_K is defined by

$$R_K = \begin{pmatrix} \cos\frac{2\pi K}{N} & -\sin\frac{2\pi K}{N} \\ \sin\frac{2\pi K}{N} & \cos\frac{2\pi K}{N} \end{pmatrix}. \tag{3.34}$$

The $(\bar{U}_\mu^{(\mathrm{A})})_{ab}$ can be diagonalized simultaneously and the diagonal elements $\lambda_\mu^{Kn\alpha}$ can be given as

$$\begin{aligned} \lambda_1^{Kn\alpha} &= \begin{cases} \mathrm{e}^{i\frac{2\pi K}{N}} & \text{for} \quad \alpha = 1 \\ \mathrm{e}^{-i\frac{2\pi K}{N}} & \text{for} \quad \alpha = 2 \end{cases} \\ \lambda_2^{Kn\alpha} &= \mathrm{e}^{-i\frac{2\pi n}{N}} \\ \lambda_i^{Kn\alpha} &= 1 \quad \text{for} \quad i = 3, 4, \cdots, 10. \end{aligned} \tag{3.35}$$

(ii) $T^a = Y^k$ and $T^b = Y^l$

$$\begin{aligned} (\bar{U}_1^{(\mathrm{A})})_{ab} &= \delta_{kl} \\ (\bar{U}_2^{(\mathrm{A})})_{ab} &= \delta_{k,\overline{l+1}} \\ (\bar{U}_i^{(\mathrm{A})})_{ab} &= \delta_{kl} \quad \text{for} \quad i = 3, 4, \cdots, 10, \end{aligned} \tag{3.36}$$

The $(\bar{U}_\mu^{(\mathrm{A})})_{ab}$ can be diagonalized simultaneously and the diagonal elements λ_μ^k can be given as

$$\begin{aligned} \lambda_1^k &= 1 \\ \lambda_2^k &= \mathrm{e}^{i\frac{2\pi k}{N}} \\ \lambda_i^k &= 1 \quad \text{for} \quad i = 3, 4, \cdots, 10, \end{aligned} \tag{3.37}$$

where $k = 1, 2, \cdots, N$. $k = N$ gives the zero mode which we always have, and we drop it by hand. The zero mode corresponds to the flat directions $\bar{U}_\mu \to \bar{U}_\mu \mathrm{e}^{i\alpha_\mu}$ for moving the D-string in arbitrary direction.

Putting all these eigenvalues into the formula (3.14), we obtain

$$
\begin{aligned}
W^{(1D-str)} &= 8\sum_{K=1}^{\frac{N-1}{2}}\sum_{n=1}^{N}\left[\log b_{Kn} - \log\left\{\frac{\kappa_{Kn}}{\epsilon^{+}_{Kn}\epsilon^{-}_{Kn}}\left(\sqrt{\frac{\epsilon^{-}_{Kn}-\mu^{-}_{Kn}}{\epsilon^{+}_{Kn}+\mu^{+}_{Kn}}}+\sqrt{\frac{\epsilon^{+}_{Kn}+\mu^{+}_{Kn}}{\epsilon^{-}_{Kn}-\mu^{-}_{Kn}}}\right)^{2}\right\}\right] \\
&\quad +4\sum_{k=1}^{N-1}\left[\log b_{k} - \log\left\{\frac{\kappa_{k}}{\epsilon^{+}_{k}\epsilon^{-}_{k}}\left(\sqrt{\frac{\epsilon^{-}_{k}-\mu^{-}_{k}}{\epsilon^{+}_{k}+\mu^{+}_{k}}}+\sqrt{\frac{\epsilon^{+}_{k}+\mu^{+}_{k}}{\epsilon^{-}_{k}-\mu^{-}_{k}}}\right)^{2}\right\}\right], \qquad (3.38)
\end{aligned}
$$

where we have defined $\mu^{\pm}_{Kn} = b_{Kn} \pm m$, $\epsilon^{\pm}_{Kn} = \sqrt{\kappa_{Kn} + (\mu^{\pm}_{Kn})^2}$ and

$$
\begin{aligned}
b_{Kn} &= 2\left[\sin^2\frac{\pi K}{N} + \sin^2\frac{\pi n}{N}\right] \\
\kappa_{Kn} &= \sin^2\frac{2\pi K}{N} + \sin^2\frac{2\pi n}{N}. \qquad (3.39)
\end{aligned}
$$

Similarly, we have defined $\mu^{\pm}_{k} = b_{k} \pm m$, $\epsilon^{\pm}_{k} = \sqrt{\kappa_{k} + (\mu^{\pm}_{k})^2}$ and

$$
\begin{aligned}
b_{k} &= 2\sin^2\frac{\pi k}{N} \\
\kappa_{k} &= \sin^2\frac{2\pi k}{N}. \qquad (3.40)
\end{aligned}
$$

The two terms in eq. (3.38) can be combined as follows by extending the region of K to $1 \le K \le N$.

$$
W^{(1D-str)} = 4{\sum_{Kn}}'\left[\log b_{Kn} - \log\left\{\frac{\kappa_{Kn}}{\epsilon^{+}_{Kn}\epsilon^{-}_{Kn}}\left(\sqrt{\frac{\epsilon^{-}_{Kn}-\mu^{-}_{Kn}}{\epsilon^{+}_{Kn}+\mu^{+}_{Kn}}}+\sqrt{\frac{\epsilon^{+}_{Kn}+\mu^{+}_{Kn}}{\epsilon^{-}_{Kn}-\mu^{-}_{Kn}}}\right)^{2}\right\}\right], \qquad (3.41)
$$

where $\sum'_{Kn}$ denotes a summation over $1 \le K, n \le N$ except for $K = n = N$. This result holds also when N is an even number.

The effective action is equal to the one we would obtain if D-instantons were broken into N fractions, and N^2 fractions made of N D-instantons were placed on the lattice sites on the surface swept by the D-string. This picture is consistent with the fact that the eigenvalues of the $\bar{U}_1$ and $\bar{U}_2$ are given by $e^{i\frac{2\pi p}{N}}$ where $p = 1, 2, \cdots, N$. Physical length of the lattice spacing is $\frac{2\pi}{Na}$, which goes to zero proportionally to a when one takes the $N \to \infty$ limit and $a \to 0$ limt with Na^2 fixed [6]. The physical extent of the space time, on the other hand, is $\frac{2\pi}{a}$, which goes to infinity at the same time.

3.3 Effective action for two parallel D-strings

Finally we consider a configuration which represents two parallel D-strings.

$$
\bar{U}_1 = \begin{pmatrix} \Gamma_1 & \\ & \Gamma_1 \end{pmatrix}
$$

$$
\begin{aligned}
\bar{U}_2 &= \begin{pmatrix} \Gamma_2 & \\ & \Gamma_2 \end{pmatrix} \\
\bar{U}_3 &= \begin{pmatrix} e^{-i\frac{\theta}{2}}\mathbf{1} & \\ & e^{i\frac{\theta}{2}}\mathbf{1} \end{pmatrix}
\end{aligned} \tag{3.42}
$$

and $\bar{U}_i = 1$ for $i = 4, \cdots, 10$. The matrices are now considered to be $2N \times 2N$. We take the generators T^a as follows.

$$
X^{Kn\alpha}_{(1)} = \begin{pmatrix} X^{Kn\alpha} & \\ & 0 \end{pmatrix}; \qquad Y^k_{(1)} = \begin{pmatrix} Y^k & \\ & 0 \end{pmatrix} \tag{3.43}
$$

$$
X^{Kn\alpha}_{(2)} = \begin{pmatrix} 0 & \\ & X^{Kn\alpha} \end{pmatrix}; \qquad Y^k_{(2)} = \begin{pmatrix} 0 & \\ & Y^k \end{pmatrix}, \tag{3.44}
$$

where $K = 1, 2, \cdots, \frac{N-1}{2}$; $n = 1, 2, \cdots, N$; $\alpha = 1, 2$ and $k = 1, 2, \cdots, N$.

$$
(Z^{Kn1})_{pq} = \frac{1}{\sqrt{2}}(\delta_{p,N+\overline{K+n}}\delta_{q,n} + \delta_{p,n}\delta_{q,N+\overline{K+n}}) \tag{3.45}
$$

$$
(Z^{Kn2})_{pq} = \frac{1}{\sqrt{2}}(-i\delta_{p,N+\overline{K+n}}\delta_{q,n} + i\delta_{p,n}\delta_{q,N+\overline{K+n}}), \tag{3.46}
$$

where $K = 1, 2, \cdots, N$ and $n = 1, 2, \cdots, N$. $\bar{U}^{(\mathrm{A})}_\mu$ is already block diagonal and has non-vanishing elements only for the following choice of a and b.

<u>(i) $T^a = X^{Kn\alpha}_{(i)}$ and $T^b = X^{Km\beta}_{(i)}$; $T^a = Y^k_{(i)}$ and $T^b = Y^l_{(i)}$</u>

These give self-energy of each D-string obtained in the previous subsection. There is a zero mode for each D-string, which corresponds to the flat directions

$$
\bar{U}_\mu \to \bar{U}_\mu \begin{pmatrix} e^{i\alpha_\mu}\mathbf{1} & \\ & e^{i\beta_\mu}\mathbf{1} \end{pmatrix} \tag{3.47}
$$

for moving the two D-strings in arbitrary direction separately. Although the result should be invariant for moving the two D-strings together in the same direction due to the $\mathrm{U}(1)^{10}$ symmetry of the model, it is not necessarily so for moving them in different directions. Among such moves we are concentrating on the one given by the θ in $\bar{U}_3$. It is straightfoward to redo the calculation including the other nine parameters in the background configuration.

<u>(ii) $T^a = Z^{Kn\alpha}$ and $T^b = Z^{Km\beta}$</u>

This gives the interaction energy between the two parallel D-strings.

$$
\begin{aligned}
(\bar{U}^{(\mathrm{A})}_1)_{ab} &= (R_K)_{\alpha\beta}\delta_{nm} \\
(\bar{U}^{(\mathrm{A})}_2)_{ab} &= \delta_{\alpha\beta}\delta_{\overline{n+1},m} \\
(\bar{U}^{(\mathrm{A})}_3)_{ab} &= R(\theta)_{\alpha\beta}\delta_{nm} \\
(\bar{U}^{(\mathrm{A})}_i)_{ab} &= \delta_{\alpha\beta}\delta_{nm} \quad \text{for} \quad i = 4, \cdots, 10,
\end{aligned} \tag{3.48}
$$

where the matrix R_K is given by eq. (3.34) and $R(\theta)$ is defined by

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{3.49}$$

The $(\bar{U}^{(\mathrm{A})}_{\mu})_{ab}$ can be diagonalized simultaneously and the diagonal elements $\lambda_{\mu}^{Kn\alpha}$ can be given as

$$\begin{aligned}
\lambda_1^{Kn\alpha} &= \begin{cases} \mathrm{e}^{i\frac{2\pi K}{N}} & \text{for} \quad \alpha = 1 \\ \mathrm{e}^{-i\frac{2\pi K}{N}} & \text{for} \quad \alpha = 2 \end{cases} \\
\lambda_2^{Kn\alpha} &= \mathrm{e}^{i\frac{2\pi n}{N}} \\
\lambda_3^{Kn\alpha} &= \begin{cases} \mathrm{e}^{i\theta} & \text{for} \quad \alpha = 1 \\ \mathrm{e}^{-i\theta} & \text{for} \quad \alpha = 2 \end{cases} \\
\lambda_i^{Kn\alpha} &= 1 \qquad \text{for} \quad i = 4, \cdots, 10.
\end{aligned} \tag{3.50}$$

Substituting the $\lambda_{\mu}^{Kn\alpha}$ into the formula (3.14), we obtain

$$\begin{aligned}
W^{(2par.D-str)} &= 2W^{(1D-str)} + W^{(int)}, \\
W^{(int)} &= 8\sum_{K,n}\Bigg[\log b_{Kn}(\theta) \\
&\quad - \log\left\{\frac{\kappa_{Kn}(\theta)}{\epsilon^{+}_{Kn}(\theta)\epsilon^{-}_{Kn}(\theta)}\left(\sqrt{\frac{\epsilon^{-}_{Kn}(\theta) - \mu^{-}_{Kn}(\theta)}{\epsilon^{+}_{Kn}(\theta) + \mu^{+}_{Kn}(\theta)}} + \sqrt{\frac{\epsilon^{+}_{Kn}(\theta) + \mu^{+}_{Kn}(\theta)}{\epsilon^{-}_{Kn}(\theta) - \mu^{-}_{Kn}(\theta)}}\right)^2\right\}\Bigg],
\end{aligned}$$

where $\mu^{\pm}_{Kn}(\theta) = b_{Kn}(\theta) \pm m$ and $\epsilon^{\pm}_{Kn}(\theta) = \sqrt{\kappa_{Kn}(\theta) + (\mu^{\pm}_{Kn}(\theta))^2}$. $b_{Kn}(\theta)$ and $\kappa_{Kn}(\theta)$ are given by

$$\begin{aligned}
b_{Kn}(\theta) &= 2\left[\sin^2\frac{\pi K}{N} + \sin^2\frac{\pi n}{N} + \sin^2\frac{\theta}{2}\right] \\
\kappa_{Kn}(\theta) &= \sin^2\frac{2\pi K}{N} + \sin^2\frac{2\pi n}{N} + \sin^2\theta.
\end{aligned} \tag{3.51}$$

When we make a physical interpretation of this result, we have to fix the physical distance between the D-strings given by $X = \frac{\theta}{a}$ and divide the effective action by the interaction time $T = \frac{2\pi}{a}$ and the length of the D-strings $L = \frac{2\pi}{a}$ to get the interaction energy between the two D-strings per unit length.

$$\varepsilon = \frac{W^{(int)}}{TL} \sim a^4 N^2 |X|^2. \tag{3.52}$$

Therefore, in the large N limit with Na^2 fixed as before, we have $\varepsilon \sim$ const.$|X|^2$, which depends on the physical distance of the two parallel D-strings. This again shows that our model is not equivalent to the IKKT model in the weak coupling limit.

4 Double scaling limit and the dynamical generation of the space time

In order to reproduce string theories from matrix models, we have to take the double scaling limit. In our model, the $\mathrm{U}(1)^{10}$ symmetry is spontaneously broken in the weak coupling limit and is expected to be restored in the strong coupling regime, which means that there must be a phase transition somewhere in between. It is natural to identify the critical point of this phase transition associated with the spontaneous breakdown of the $\mathrm{U}(1)^{10}$ symmetry as the place where we take the double scaling limit. Let us consider approaching the critical point from the strong coupling regime. Since it is natural to relate our model to the IKKT model through $U_\mu = \mathrm{e}^{iaA_\mu}$, we obtain to the leading order of a as

$$S = -\frac{1}{2} N\beta a^4 \mathrm{tr}[A_\mu, A_\nu]^2 + O(a^6). \tag{4.1}$$

We therefore have $\frac{1}{2g^2} = N\beta a^4$. In Ref. [6], it has been shown through the study of Schwinger-Dyson equation that the double scaling limit is given by fixing

$$\begin{aligned} g^2 N &= \alpha'^2 & (4.2)\\ Na^2 &= \frac{1}{g_{str}\alpha'}, & (4.3) \end{aligned}$$

when one takes the large N limit, where g is the coupling constant in the action (2.1). Translating this into our model, we have

$$\begin{aligned} \frac{\beta}{N^2} &= \frac{1}{2} g_{str}^2 & (4.4)\\ \beta a^4 &= \frac{1}{2\alpha'^2}, & (4.5) \end{aligned}$$

which means that β has to be sent to infinity as $\beta \sim N^2$. In order to approach the critical point in the double scaling limit, the pseudo-critical point $\beta_c(N)$ for finite N must behave in the large N limit as $\beta_c(N) \sim N^2$. This gives a nontrivial test of our scenario which could be checked by numerical simulation in principle. This rather unusual shift of the critical point is not very unlikely, considering the fact that actually the $\mathrm{U}(1)^{10}$ symmetry is much more mildly broken in the weak coupling limit than in the purely bosonic case where there exists an attractive logarithmic potential among the D-objects and the spontaneous breakdown of the $\mathrm{U}(1)^D$ symmetry occurs at finite β in the large N limit. The nontrivial cancellation of the logarithmic potentials from the bosonic part and the fermionic part is actually due to the supersymmetry at $U_\mu \sim 1$, where our model reduces to the IKKT model. In this sense,

although supersymmetry is not manifest in our model, it plays an important role in the above argument.

The results in the weak coupling limit for our model are different from the ones for the IKKT model, which have been regarded as an evidence that the IKKT model reproduces the massless spectrum of the type IIB superstring theory in Ref. [3]. We point out, however, that the calculation in the weak coupling limit has nothing to do with the double scaling limit *a priori*. In this sense, we regard the success of the IKKT model in the weak coupling limit as accidental. More to the point, we consider that the fact that the $\mathrm{U}(1)^{10}$ symmetry is not spontaneously broken in the IKKT model even in the weak coupling limit suggests that it is never broken spontaneously throughout the whole region of the coupling constant, which we suspect is the reason why the properties that should be attributed to the double scaling limit have been obtained even in the weak coupling limit. In the present model, on the other hand, if the pseudo-crititcal point $\beta_c(N)$ for finite N goes to infinity in the large N limit as is claimed above, the $\beta \to \infty$ limit and the $N \to \infty$ limit is not commutable and the result should depend on how one takes the two limits. Therefore the results in the weak coupling limit do not necessarily reflect the properties in the double scaling limit.

The spontaneous breakdown of the $\mathrm{U}(1)^{10}$ symmetry in the weak coupling limit is actually welcome when we consider the dynamical generation of the four-dimensional space time in the double scaling limit. Let us speculate on how we could hope for obtaining the four-dimensional space time through our model in the double scaling limit. Since the phase transition is associated with spontaneous breakdown of the $\mathrm{U}(1)^{10}$ symmetry, which is interpreted as the translational invariance of the 10D space time, naturally we would have a space time of dimension between 0 and 10 in the double scaling limit. We give a hand-waving argument for obtaining the space-time dimension four in the double scaling limit for sufficiently small g_{str}. Here, planar diagrams dominate and the equivalence of the reduced model and the large N gauge theory is expected due to the argument of Eguchi-Kawai [14]. Rougly speaking, if $\mathrm{U}(1)^{10}$ symmetry is broken down to $\mathrm{U}(1)^D$, the reduced model is equivalent to D-dimensional gauge theory. We pay attention to the fact that the critical dimension of gauge theories is four. Namely, we can obtain a nontrivial continuum theory by approaching the Gaussian fixed point only when the dimension of the space time is equal to or less than four. This suggests that the D of the remaining $\mathrm{U}(1)^D$ symmetry should be equal to or less than four [4]. We do not have any plausible reasoning to exclude the possibility of the

[4] In Ref. [17], an essential difference in the phase diagram between the one-site model with $\mathrm{U}(1)^4$ symmetry

space-time dimenion turning out to be less than four, but naively we may expect that the critical value "four" has a special meaning.

5 Summary and Future Prospects

In this paper, we proposed a unitary model as a regularization of the IKKT model, which is considered to give a nonperturbative definition of the type IIB superstring theory. Our model preserves manifest $U(1)^{10}$ symmetry, which corresponds to the ten-dimensional translational invariance. On the other hand, the $\mathcal{N}=2$ supersymmetry as well as the continuous 10D Lorentz invariance is expected to be restored only in the double scaling limit.

One-loop calculation of the effective action around some typical BPS-saturated states has been performed. The corresponding calculation in the IKKT model is done by formally taking $N=\infty$ and $a=0$ from the beginning, while we have done our calculation for finite N and non-vanishing a. The results for our model differ from the ones for the IKKT model. Above all, the $U(1)^{10}$ symmetry is spontaneously broken in our model. We argued, however, that this is not a problem itself. Rather, the phase transition associated with the $U(1)^{10}$ symmetry breakdown provides a natural place to take the double scaling limit. The crucial feature necessary for our model to work is that the pseudo-critical coupling $\beta_c(N)$ shifts as N^2 when one takes N to the infinity. The spontaneous $U(1)^{10}$ symmetry breakdown is also welcome for a natural explanation of the dynamical generation of the space time in the double scaling limit. We gave a hand-waving argument that for sufficiently small g_{str}, the space-time dimension is likely to be four.

Monte Carlo simulation of our model and the IKKT model is possible in principle. We hope that it will show whether our considerations are correct or not. It would be interesting if we could make some approximation and extract some nonperturbative physics analytically, just as in Ref. [1] the qualitative understanding of the quark confinement has been given by the strong coupling expansion.

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and the one with $U(1)^6$ symmetry has been revealed for purely bosonic case through Monte Carlo simulation.

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